Solutions to JEE Advanced Booster Test - 5 | 2024 | Code A

[PHYSICS]

SECTION 1

1.(C) Let linear density y, $\lambda = a\left(1+\frac{x}{L}\right)$, where a is some positive constant

$$dm = \lambda dx = a\left(1+\frac{x}{L}\right)dx\,; \qquad M = \int dm = a\int_0^L \left(1+\frac{x}{L}\right)dx$$

$$\Rightarrow \qquad M = a\left(L+\frac{L}{2}\right) = \frac{3}{2}aL \qquad \text{or} \qquad a = \frac{2M}{3L}$$

$$I = \int x^2 dm = a\int_0^L x^2\left(1+\frac{x}{L}\right)dx = a\left\{\frac{L^3}{3}+\frac{L^3}{4}\right\}$$

$$= \frac{7aL^3}{12} = \frac{7}{12}\cdot\frac{2M}{3L}\cdot L^3 \qquad = \frac{7}{18}ML^2$$

2.(C) $N_1\mu R + N_2\mu R = \frac{1}{2}mR^2\alpha$

$$\Rightarrow \qquad \frac{mg\,\mu}{1+\mu^2} + \frac{mg\,\mu^2}{1+\mu^2} = \frac{m\,R\,\alpha}{2} \qquad \Rightarrow \qquad \alpha = \frac{2g\mu(1+\mu)}{R\left(1+\mu^2\right)}$$

3.(C) By momentum conservation: (In horizontal direction)

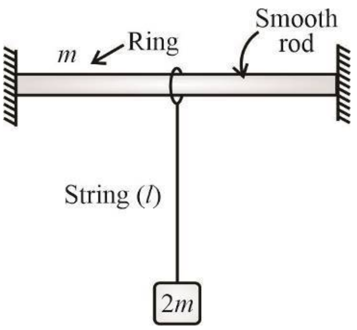
$mv_1 = 2mv_2$

and by energy conservation:

$$2mgl = \frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2$$

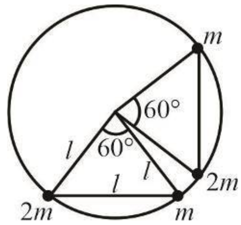
$$\Rightarrow \qquad 2gl = \frac{1}{2}v_1^2 + \left(\frac{v_1}{2}\right)^2$$

$$\Rightarrow \qquad 2gl = \frac{3}{4}v_1^2 \qquad \Rightarrow \qquad v_1 = \sqrt{\frac{8gl}{3}}$$



4.(B) The speed given to $2m$ will also be possessed by m.

$\therefore$ KE in horizontal position gets converted in PE in vertical position



$\frac{1}{2}2mv^2 + \frac{1}{2}mv^2 =$ change in PE in vertical position

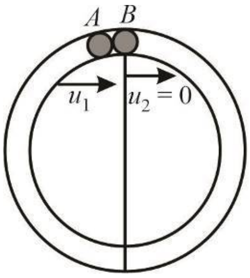
$$\Delta PE = 2mg[l\cos 30° - l\cos 60°] + mg\left[l\cos 30° + \frac{l}{2}\right]$$

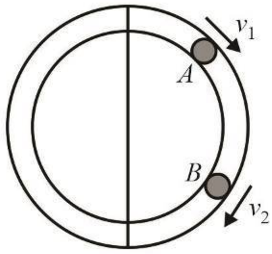
$$2mg\left[\frac{l\sqrt{3}}{2} - \frac{l}{2}\right] + mg\left[\frac{l\sqrt{3}}{2} + \frac{l}{2}\right] \qquad \therefore \qquad mgl\,[\sqrt{3}-1] - mgl\left[\frac{\sqrt{3}+1}{2}\right]$$

$$= mgl\left[\sqrt{3}-1+\frac{\sqrt{3}}{2}+\frac{1}{2}\right] = mgl\left[\frac{3\sqrt{3}}{2}-\frac{1}{2}\right]$$

$$K.E. = \frac{1}{2}3mv^2 = mgl\left[\frac{3\sqrt{3}-1}{2}\right] \qquad \therefore \qquad v = \sqrt{\left(\frac{3\sqrt{3}-1}{3}\right)gl}$$

5.(D) $T_1 = \frac{\pi R}{u_1}$ ... (i)





$\frac{v_2 - v_1}{u_1} = e \Rightarrow v_2 - v_1 = eu_1$

$t = T_1$

Time taken to collide A and B again is

$$T_2 - T_1 = \frac{2\pi R}{v_2 - v_1}$$

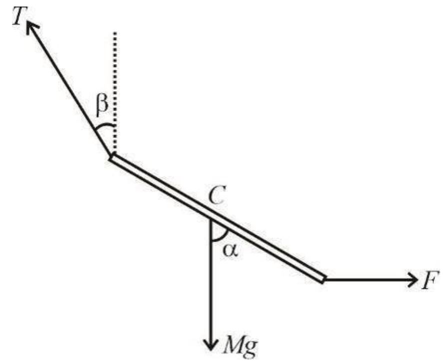
$\Rightarrow \qquad T_2 - T_1 = \frac{2\pi R}{eu_1}$ ... (ii)

Dividing (ii) by (i), we get $\frac{T_2}{T_1} = \frac{2+e}{e}$

6.(C) Horizontal and vertical equilibrium of the stick requires

$F = T\sin\beta \Rightarrow Mg = T\sin\beta$ ... (i)

and $\qquad mg = T\cos\beta$ ... (ii)



Solving (i) and (ii)

$\tan\beta = 1 \Rightarrow \beta = 45°$; $\qquad T = \sqrt{2}Mg$

Rotational equilibrium requires torque about C to be zero $(\tau_c = 0)$

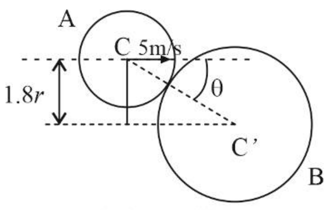
$$T\cos\beta\frac{l}{2}\sin\alpha - T\sin\beta\frac{l}{2}\cos\alpha = F\frac{l}{2}\cos\alpha$$

$$T\sin\alpha - T\cos\alpha = \sqrt{2}F\cos\alpha \qquad \left[\because \qquad \cos\beta = \sin\beta = \frac{1}{\sqrt{2}}\right]$$

$$\sqrt{2}\,Mg(\sin\alpha - \cos\alpha) = \sqrt{2}\,Mg\cdot\cos\alpha \qquad \therefore \qquad \tan\alpha = 2 \qquad \Rightarrow \qquad \alpha = \tan^{-1}[2]$$

7.(BD) $\sin\theta = \frac{1.8r}{3r} = \frac{3}{5}$

Velocity of centre of A along common normal $cc' = 5\cos\theta = 4\ m/s$



Just before collision

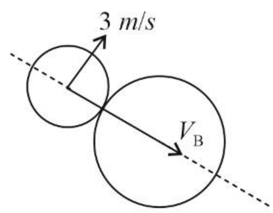
Component of velocity perpendicular to $cc' = 5\sin\theta = 3\ m/s$

After collision, velocity of A along common normal changes but the component perpendicular to it remains same

Also B will gain velocity only along the common normal.

It is observed that A and B move perpendicular to each other, so velocity of A along common normal should become zero.

$\therefore \qquad V_A = 3\ m/s$



Just after collision

To find V_B, apply momentum conservation

$m_A(4) = m_B V_B \Rightarrow V_B = 1\ m/s$; $\quad e = \dfrac{1-0}{4-0} = 0.25$

8.(AC) Torque about the pivot

$\tau = 3mgl = I\alpha$ where $I = ml^2 + m(21)^2 = 5ml^2$

This gives $\alpha = \dfrac{3g}{5l} = \dfrac{3\times10}{5\times2} = 3\,rad/s^2$

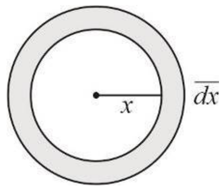
Also, $\quad (mg+mg) - R = 2ma_{cm} = ma_1 + ma_2$

Substituting $a_1 = l\alpha$ and $a_2 = 2l\alpha$ we have $R = m(2g - 3l\alpha)$

$= 2(2\times10 - 3\times2\times3) = 2(20-18) = 4N$

9.(AB) Frictional force on elemental disc

$= \mu\{2\pi x\, dx\}\cdot\{\rho\}g \qquad$ and $\qquad \rho = \dfrac{M}{\pi R^2}$



Thus, $\quad$ Torque $\displaystyle\int_0^R \frac{2Mg\mu}{R^2}(x)\,(dx)\times x = \frac{2}{3}\mu MgR$

$\therefore \qquad \alpha \dfrac{\text{Torque}}{I} = \dfrac{(2/3)(\mu)(M)(g)(R)}{(MR^2/2)} \qquad \Rightarrow \qquad \alpha = \dfrac{4}{3}\dfrac{\mu g}{R}$

10.(AD)

(A) $\quad$ By conservation of linear momentum $v = \dfrac{u}{3}$

(B) $\quad \displaystyle\int Tdt = \frac{mu}{3}$

(C) $\quad \displaystyle\int Ndt = m\left(\frac{u}{3} - u\right) = \frac{-2mu}{3}$

(D) $\quad \displaystyle\int (N-T)\,dt \Rightarrow \int Ndt - \int Tdt = \frac{mu}{3} \qquad \Rightarrow \qquad \int Ndt = \frac{2mu}{3}$

PARAGRAPH FOR Q-11 & 12

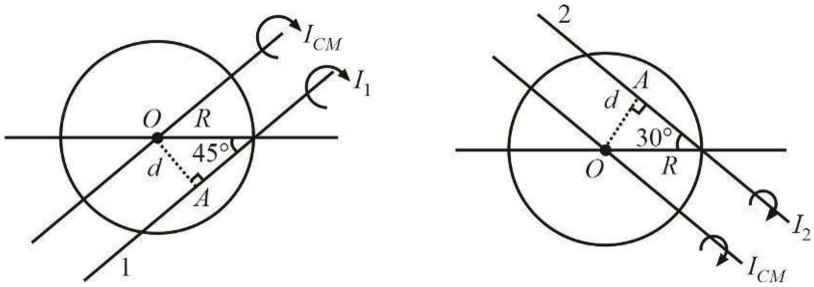
11.(A)

12.(D) Conceptual

SECTION 2

1.(6) $\quad I_1 = I_{CM} + Md^2 = \dfrac{MR^2}{4} + M\left(\dfrac{R}{\sqrt{2}}\right)^2$

$= \dfrac{3}{4}MR^2$



$$I_2 = I_{CM} + Md^2 = \frac{MR^2}{4} + M\left(\frac{R}{2}\right)^2 = \frac{MR^2}{2} \quad \therefore \qquad \frac{I_1}{I_2} - \frac{3}{4}\times\frac{2}{1} = \frac{3}{2}$$

2.(18) $-2.2 = 10t - 5t^2$

$5t^2 - 10t - 2.2 = 0$

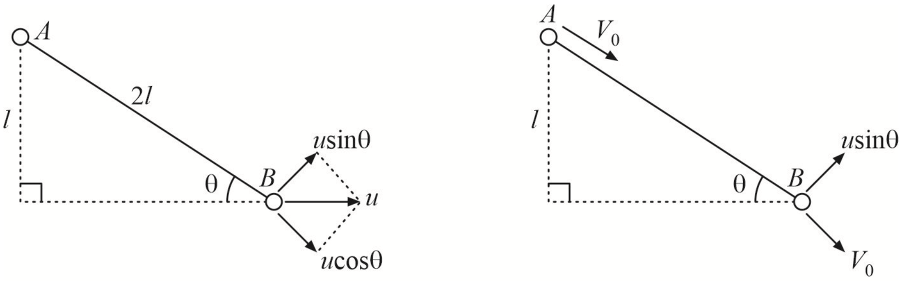
$t = \frac{10 \pm \sqrt{144}}{10} = \frac{22}{10} = 2.2\,\text{sec}$

$x = 10 \times 1.8 = 18\,m$

3.(10) $m(10l - x) = Mx$

$x = \frac{m \times 10l}{M + m}$

4.(6)



Just before string become taut

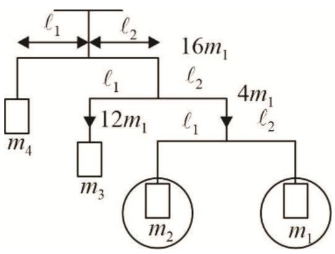
After string becomes taut

$\sin\theta = \frac{l}{2l} = \frac{1}{2} \qquad \Rightarrow \qquad \theta = 30°$

Conservation of momentum along the string $mu\cos\theta = mv_0 + mv_0$

$\therefore \qquad V_0 = \frac{u\cos\theta}{2} = \frac{8\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2} = 6$

5.(4) $l_1 = \frac{l_2}{3}, \qquad l_2 = 3l_1$



$m_1 l_2 = m_2 l_1$

$m_2 = \frac{m_1 l_2}{l_1} = 3m_1, \; 4m_1 l_2 = m_3 l_1$

$m_3 = 12m_1, \frac{m_3}{m_2} = \frac{12m_1}{3m_1} = 4$

6.(17) The given body can be considered the superposition of:

(a) a disc of radius $2R$ and mass per unit area σ_0, and

(b) a disc of radius R and mass per unit area σ_0

Now, moment of inertia of a disc of radius R about its diameter,

$$I = \frac{1}{4} mR^2 = \frac{1}{4}(\sigma_0 \pi R^2)R^2 = \frac{1}{4}\pi\sigma_0 R^4$$

Therefore, total moment of inertia of the given body about a diameter,

$$I_{Total} = \frac{1}{4}\pi\sigma_0 R^4 + \frac{1}{4}\pi\sigma_0 (2R)^4 = \frac{17}{4}\pi\sigma_0 R^4$$

So, the kinetic energy of the body about the diameter is.

$$K = \frac{1}{2} I_{Total}\omega^2 = \frac{17}{8}\pi\sigma_0 R^4\omega^2$$

[CHEMISTRY]

SECTION 1

1.(A) $\underbrace{P_4(s) \xrightarrow{59} P_4(g) \xrightarrow{4x} 4P(g)}_{1265}$

59 + 6x = 1265

6x = 1265 – 59

$x = \frac{1206}{6} = 201\,\text{kJ/mol}$

2.(D) $\Delta H = \Delta U + \Delta n_g RT$

$\Delta n_g = +\frac{1}{2}$

$\Delta H = \Delta U + \left(\frac{1}{2}\right)RT$

Thus $\Delta H > \Delta U$

3.(B) $CH_4 = \text{a ml}$

$C_2H_4 \rightarrow (30-a)\text{ml}$

$\frac{a}{30-a} = \frac{x}{y}$

$\underset{\text{a ml}}{CH_4} + O_2 \rightarrow \underset{\text{a ml}}{CO_2} + H_2O$

$\underset{(30-a)}{C_2H_4} + O_2 \rightarrow \underset{2(30-a)}{2CO_2} + H_2O$

$a + 60 - 2a = 40$

$\Rightarrow \quad 60 - a = 40 \quad \Rightarrow \quad a = 20\text{ ml}$, volume of CH_4

Volume of $C_2H_4 = 10\text{ ml}$

$\frac{X}{Y} = \frac{2}{1}$

Now, in Y : X ratio, $CH_4 = 10\text{ ml}$, $C_2H_4 = 20\text{ ml}$

$\underset{10\text{ ml}}{CH_4} + O_2 \rightarrow \underset{10\text{ ml}}{CO_2}$

$\underset{20\text{ ml}}{C_2H_4} + O_2 \rightarrow \underset{40\text{ ml}}{2CO_2}$

Total volume of $CO_2 = 50\text{ ml}$

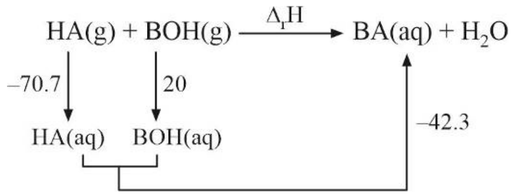
4.(C) Only 4^{th} is formation reaction.

5.(A) $-152 = 3[-119] - x \quad \Rightarrow \quad x = -3(119) + 152 = -205$

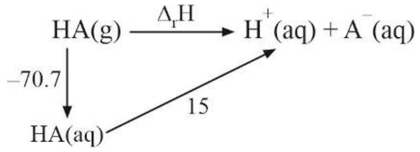
6.(ABC)

(A) $\Delta_r H = 15 + 0 - 57.3 = -42.3 \text{ kJ}$

(B)



(C)



$\Delta_r H = -70.7 + 15 = -55.7$

(D) $B^+(aq) + OH^-(aq) \longrightarrow BOH(aq)$

$\Delta_r H = 0$

7.(AD) $\dfrac{2a}{2} + \dfrac{a}{2} - 2a = -100 \quad \Rightarrow \quad a = 200$

$\therefore \quad 2a = 400 \text{ kJ/mol} \quad$ or $\quad 95.6$ Kcal/mol

8.(AC) (A) Use PV = nRT, $P = \dfrac{2 \times 0.0821 \times 300}{8.21} = 6 \text{atm}$

(B) $P = \dfrac{2 \times 0.0821 \times 400}{8.21} = 8 \text{atm}$

(C) For isobaric condition volume changes $\dfrac{V_2}{T_2} = \dfrac{V_1}{T_1}$

$\dfrac{8.21}{300} = \dfrac{V_2}{600}; V_2 = 16.422$

(D) $n_1 + n_2 = n_1' + n_2'$

$2 + 0 = P_f \dfrac{(8.21)}{R(300)} + P_f \dfrac{(8.21)}{R(150)}$

$\therefore \quad P_f = 2 \text{atm}$

9.(AB) Gas can be liquefied at critical condition, at Boyle's temperature real gas behaves like ideal gas at low pressure only.

10.(ABC)

(A) $V_t = V_0 + \dfrac{t°C}{273} V_0$; intercept is not zero

(B) $V_T = KT$; intercept is zero

(C) $\frac{V}{T} = K$, plot of V/T versus T is linear with zero slope

PARAGRAPH FOR Q-11 & 12

11.(B) $n_A = \frac{pV}{RT} = \frac{910}{760} \times \frac{2.5}{0.082 \times 300} = 0.122 \text{ mol}$

12.(C) Initial: $A : n = 0.122,\ V = 2.5L,\ T = 27°C$

$B : n = 0.488,\ V = 10L,\ T = 27°C$

Let, finally A has n_A and B has n_B moles

$\Rightarrow \quad \left(\frac{V}{nT}\right)_A = \left(\frac{V}{nT}\right)_B \qquad \therefore \qquad$ p will be same in both flask

$\Rightarrow \quad \frac{2.5}{n_A \times 300} = \frac{10}{n_B \times 400} \quad \Rightarrow \quad 3n_A = n_B$

Also, $\quad n_A + n_B = 0.122 + 0.488 = 0.61 \quad \Rightarrow \quad 4n_A = 0.61,\ n_A = 0.1525$

Hence, $\frac{p_A}{p_{A,\,\text{final}}} = \frac{0.122}{0.1525} \quad \Rightarrow \quad p_A(\text{final}) = \frac{910 \times 0.1525}{0.122} = 1137.5\,\text{mm}$

SECTION 2

1.(2.8) $\frac{r_A}{r_B} = \sqrt{\frac{M_B}{M_A}} = \frac{2}{1}$ and $\frac{(V_{rms})_A}{(V_{rms})_B} = \sqrt{\frac{T_A}{T_B} \times \frac{M_B}{M_A}} = \sqrt{\frac{2}{1} \times \frac{4}{1}} = 2\sqrt{2} : 1$

2.(5) $2P_2(s) \longrightarrow P_4(s)$

$\Delta H = \Delta H_{comb.}(\text{Reactant}) - \Delta H_{comb.}(\text{Product}) = -4 \times 8.65 - (-9.9) \times 4 = 5 \text{ kJ}$

3.(4) $n = 4 : \underset{1-0.52}{A_n(g)} \rightleftharpoons \underset{0.52\,n}{nA(g)}$

$\Rightarrow \quad \overline{M} = \frac{32n}{0.48 + 0.52n} = 50 \Rightarrow n = 4$ [average molar mass of the mixture $= (1.25)^2 32 = 50$]

4.(9) $T_b = \frac{a}{Rb} = \frac{0.36}{0.08 \times 0.5} = 9\,\text{Kelvin}$

5.(35) Number of milli-equivalent of $Ca(OH)_2 = 50 \times 2 \times 0.01 = 1.00$

Number of milli-equivalents of HCl = 25 × 0.01 = 0.25

0.25 milli-equivalent of HCl will be neutralized by 0.25 milli-equivalents of $Ca(OH)_2$

1 equivalent corresponds to –140.0 kcal

Therefore, 0.25×10^{-3} equivalent corresponds to $-140.0 \times 0.25 \times 10^{-3}\,\text{kcal} = -35\,\text{cal}$

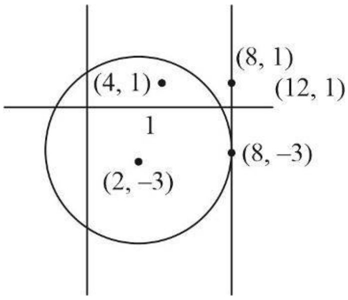
6.(20) As $V_{mp} = \sqrt{\frac{2RT}{M°}}$

$$\Rightarrow \qquad M^\circ = \frac{2RT}{V_{mp}^2} = \frac{2\times 8.3\times 300}{500\times 500}\text{kg mol}^{-1} = \frac{2\times 8.3\times 300}{500\times 500}\times 10^3\text{g mol}^{-1} = \frac{2\times 8.3\times 30}{25} = 19.92\text{ g mol}^{-1}$$

[MATHEMATICS]

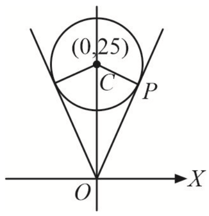
SECTION 1

1.(A) $|z-2+3i|=6$ is a circle with centre (2, –3) and radius 6 and $|z-4-i|=|z-12-i|$ is the perpendicular bisector of (4, 1) and (12, 1) is a line parallel to imaginary axis.



The line is tangent to circle at complex number (8 – 3i).
Hence only one complex number satisfying the above

2.(D) The required complex number is point of contact P as shown in the figure. C(0, 25) is the centre of the circle and radius is 15



Now, $\quad |z| = OP = \sqrt{OC^2 - PC^2}$

$= \sqrt{625-225} = 20$

$amp(z) = \theta = \angle XOP = \angle OCP$

$\therefore \qquad \cos\theta = \frac{PC}{OC} = \frac{15}{25} = \frac{3}{5}$

and $\qquad \sin\theta = \frac{OP}{OC} = \frac{20}{25} = \frac{4}{5}$

$\therefore \qquad z = 20\left(\frac{3}{5}+\frac{4}{5}i\right) = 12+16i$

3.(C) Let $A(0, a)$ and $B(b, 2b)$

Coordinate of P is $\left(\frac{2a}{5}, \frac{4a}{5}\right)$

Coordinate of Q is $(0, 2b)$

Equation of AB is $y - a - \left(2-\frac{a}{b}\right)x = 0$

$\because \qquad$ It passes through (1, 3) hence $a = \frac{b}{b-1}$

Also equation of PQ is $y - 2b = \left(2-\frac{5b}{a}\right)x$

i.e. $y - 7x + b(5x-2) = 0$

$\therefore \qquad$ Fixed point is $\left(\frac{2}{5}, \frac{14}{5}\right)$

4.(C) Let $z = r(\cos\theta + i\sin\theta)$

Then the equation $r^2 + \frac{4}{r^2} - 2.2\cos 2\theta - 16 = 0$

$r^4 - 4r^2(\cos 2\theta + 4) + 4 = 0$

By solving we get

For maximum value of r, $\cos 2\theta = 1$

$r = 2 + \sqrt{6}$

5.(B) Here $z - 1 = e^{i\theta}$

$\Rightarrow \quad z = 2\cos^2\frac{\theta}{2} + 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2\cos\frac{\theta}{2}e^{i\theta/2}$

and $\quad z - 2 = z - 1 - 1 = e^{i\theta} - 1 = -1 + e^{i\theta} = -2\sin^2\frac{\theta}{2} + 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2i\sin\frac{\theta}{2}e^{i\theta/2}$

Hence $\quad 2i\sin\frac{\theta}{2}e^{i\theta/2} = k \cdot 2\cos\frac{\theta}{2}e^{i\theta/2}\tan\frac{\theta}{2}$

$\Rightarrow \quad k = i$

6.(BCD)

$(3a + 4b)(3a - 2b) = 0$ and $(3a - 2b)(a + b) = 0$

$\Rightarrow \quad 3a = 2b$

7.(AD) Let $P(2, -1)$ goes 2 units along $x + y = 1$ upto A and 5 units along $x - 2y = 4$ upto B

Slope of $PA = -1 = \tan 135°$, slope of $PB = \frac{1}{2} = \tan\theta$

$\therefore \quad \sin\theta = \frac{1}{\sqrt{5}}, \ \cos\theta = \frac{2}{\sqrt{5}}$

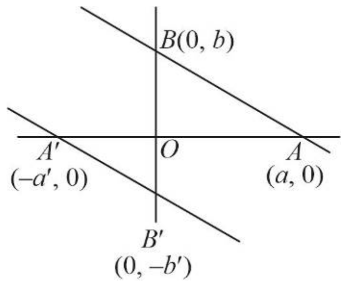
$\therefore \quad A \equiv (x_1 + r\cos 135°, y_1 + r\sin 135°) = \left(2 + 2\times\frac{-1}{\sqrt{2}}, -1 + 2\times\frac{1}{\sqrt{2}}\right) = (2 - \sqrt{2}, \sqrt{2} - 1)$

$B \equiv (x_1 + r\cos\theta, \ y_1 + r\sin\theta) = \left(2 + 5\times\frac{2}{\sqrt{5}}, -1 + \frac{5}{\sqrt{5}}\right) = (2\sqrt{5} + 2, \ \sqrt{5} - 1)$

8.(BC) If A, B, A', B' are concyclic then

$\Rightarrow \quad (OA)(OA') = (OB)(OB')$

$\Rightarrow \quad bb' = aa'$



For $\Delta ABA'$ orthocenter lie on line OB

$\therefore$ Let the co-ordinate of orthocenter is $H(0, k)$ then

Slope of $A'H = \left(\frac{k}{a'}\right)$

Slope of $AB = \left(-\frac{b}{a}\right)$

$\therefore \quad \left(\frac{k}{a'}\right)\left(\frac{-b}{a}\right) = -1 \qquad$ or $\qquad k = \frac{aa'}{b} = b'$

$\therefore$ co-ordinates of orthocenter are $(0, b')$

or $\quad \left(0, \frac{aa'}{b}\right)$

9.(AB) $|z - i\,\mathrm{Re}(z)| = |z - \mathrm{IM}(z)|$

Let $\quad z = x + iy$, then $|x + iy - ix| = |x + iy - y|$

i.e. $\quad x^2 + (y - x)^2 = (x - y)^2 + y^2$

i.e. $\quad x^2 = y^2$

i.e. $y = \pm x$

10.(AC)

Let $z = \alpha$ (real root)

$$\alpha^3 + (3+i)\alpha^2 - 3\alpha - (m+i) = 0$$

$$\alpha^3 + 3\alpha^2 - 3\alpha - m = 0$$

$\Rightarrow \quad \alpha^2 - 1 = 0 \quad \therefore \quad m = 1,\ 5 \quad \Rightarrow \quad \alpha = \pm 1$

PARAGRAPH FOR Q-11 & 12

11.(B) Let C be the reflection of A in $L \equiv x + y = 0$

$\Rightarrow \quad C \equiv (-2, -1)$

Now, $AM + BM = CM + BM$

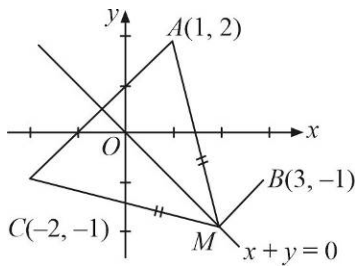
Which is minimum if B, M and C are collinear

Now, equation of BC is

$$(y+1) = 0(x+2)$$

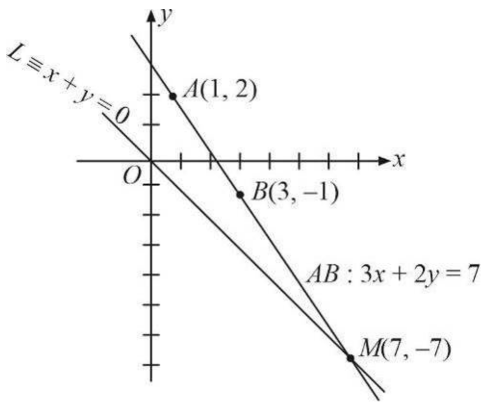
$\Rightarrow \quad y + 1 = 0 \quad \ldots \text{(i)}$

$\therefore$ On solving equation (1) with $x + y = 0$, we get $M \equiv (1, -1)$



Hence the reflection of M in the line $x = y$ is $(-1, 1)$

12.(D) We have $|AM - BM| \le AB$



$\therefore$ For $|AM - BM|$ to be maximum, points A, B and M must be collinear

Now, equation of AB is $(y-2) = -\dfrac{3}{2}(x-1)$

$\Rightarrow \quad 3x + 2y = 7 \quad \ldots \text{(i)}$

$\therefore$ On solving equation (i) with $x + y = 0$, we get $M \equiv (7, -7)$

Hence distance of $M(7, -7)$ from $N(1,\ 1) = MN = \sqrt{(7-1)^2 + (-7-1)^2}$

$= \sqrt{36 + 64} = 10$ units

SECTION 2

1.(15) We have $(z_1 + 2)(z_2 + 2)(z_3 + 2) = z_1 z_2 z_3 + 2(z_1 z_2 + z_1 z_3 + z_2 z_3) + 4(z_1 + z_2 + z_3) + 8$

$$= 13 + 2(z_1 z_2 + z_1 z_3 + z_2 z_3) \quad \ldots \text{(i)}$$

Now, $z_1 z_2 + z_2 z_3 + z_3 z_1 = z_1 z_2 z_3 \left(\dfrac{1}{z_1} + \dfrac{1}{z_2} + \dfrac{1}{z_3} \right)$

$$\Rightarrow \quad z_1z_2 + z_2z_3 + z_3z_1 = \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}$$

Now as, $|z_1| = 1 \quad \Rightarrow \quad z_1\bar{z}_1 = 1 \quad \Rightarrow \quad \frac{1}{z_1} = \bar{z}_1$

So, $\quad z_1z_2 + z_2z_3 + z_3z_1 = \bar{z}_1 + \bar{z}_2 + \bar{z}_3 = \overline{z_1 + z_2 + z_3} = \bar{1} = 1$

Hence from (i) we get $(z_1+2)(z_2+2)(z_3+2) = 13 + 2(1) = 15$

2.(6) $|z-3| \le |z-1| \Rightarrow z + \bar{z} \ge 4 \Rightarrow x \ge 2 \qquad (\because\ z + \bar{z} = 2x)$

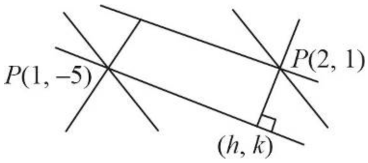
$|z-3| \le |z-5| \Rightarrow z + \bar{z} \le 8 \Rightarrow x \le 4$

$|z-i| \le |z+i| \Rightarrow i(z-\bar{z}) \le 0 \Rightarrow y \ge 0 \qquad (\because\ z - \bar{z} = 2iy)$

$|z-i| \le |z-5i| \Rightarrow i(z-\bar{z}) \ge -6 \Rightarrow y \le 3$

Clearly the region is a rectangle of area = 2 × 3 = 6

3.(2)



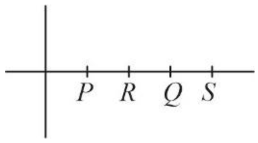
We have $\left(\frac{k+5}{h-1}\right)\left(\frac{k-1}{h-2}\right) = -1$

$\therefore \quad$ Locus of (h, k) is $x^2 + y^2 - 3x + 4y - 3 = 0$

4.(2) Let $\quad P(x_1, 0), Q(x_2, 0), R(x_3, 0)$ and $S(x_4, 0)$

$$x_1 + x_2 = \frac{-2b_1}{a_1},\ x_1x_2 = \frac{c_1}{a_1}$$

$$x_3 + x_4 = \frac{-2b_2}{a_2},\ x_3x_4 = \frac{c_2}{a_2}$$



Let R divides PQ internally in ratio $k : 1$ and S divides externally in $k : 1$

$$\frac{kx_2 + x_1}{k+1} = x_3,\ \frac{kx_2 - x_1}{k-1} = x_4$$

$$\Rightarrow \quad kx_2 + x_1 = kx_3 + x_3 \text{ and } kx_2 - x_1 = kx_4 - x_4$$

$$\Rightarrow \quad k = \frac{(x_3 - x_1)}{x_2 - x_3} \text{ and } k = \frac{x_1 - x_4}{x_2 - x_4}$$

$$\Rightarrow \quad \frac{x_3 - x_1}{x_2 - x_3} = \frac{x_1 - x_4}{x_2 - x_4}$$

$$\Rightarrow \quad x_2x_3 - x_1x_2 - x_3x_4 + x_1x_4 = x_1x_2 - x_2x_4 - x_1x_3 + x_3x_4$$

$$\Rightarrow \quad -2(x_1x_2 + x_3x_4) = -x_3(x_1 + x_2) - x_4(x_1 + x_2)$$

$$\Rightarrow \quad 2(x_1x_2 + x_3x_4) = (x_1 + x_2)(x_3 + x_4)$$

$$\Rightarrow \quad 2\left(\frac{c_1}{a_1} + \frac{c_2}{a_2}\right) = \frac{2b_1}{a_1} \cdot \frac{2b_2}{a_2}$$

$$\Rightarrow \quad a_1c_2 + a_2c_1 = 2b_1b_2$$

5.(100)

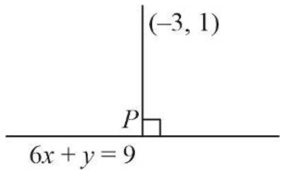
$6x + y = 9$

Equation of perpendicular line from (–3, 1)

$$y-1=\frac{1}{6}(x+3)$$

$\Rightarrow \quad x-6y+9=0 \quad \ldots \text{(i)}$

$6x+y-9=0 \quad \ldots \text{(ii)}$



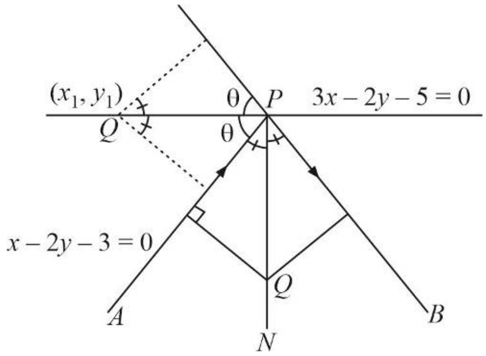
On solving (i) and (ii), we get

$$y=\frac{63}{37}=\frac{a}{b}$$

$$a+b=100$$

6.(62) Equation of the reflected ray BP can be taken as

$x-2y-3+\lambda(3x-2y-5)=0 \quad \ldots \text{(i)}$



Now perpendicular from $Q(x_1, y_1)$ on (i) and $x-2y-3=0$ should be equal

$$\left|\frac{x_1-2y_1-3}{\sqrt{5}}\right|=\left|\frac{x_1-2y_1-3+\lambda(3x_1-2y_1-5)}{\sqrt{(3\lambda+1)^2+4(\lambda+1)^2}}\right|$$

or $\quad (3\lambda+1)^2+4(\lambda+1)^2=5$

$$9\lambda^2+1+6\lambda+4\lambda^2+8\lambda+4=5$$

$13\lambda^2+14\lambda=0 \quad \Rightarrow \quad \lambda=-\frac{14}{13} \quad$ (as $\lambda \neq 0$)

$\therefore \quad$ Required equation is

$$x-2y-3-\frac{14}{13}(3x-2y-5)=0$$

$\Rightarrow \quad 29x-2y=31$